SSD-TDR-63-130

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REPORT NO. 63-4..
TDR-169(3250-43)TN-4

408 761

The Influence of Random Phase Errors
on the Angular Resolution of
Synthetic Aperture Radar Systems

20 JUNE 1963

Prepared by JEAN A. DEVELET, JR.
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LABORATORIES DIVISION • AEROSPACE CORPORATION

THE INFLUENCE OF RANDOM PHASE ERRORS
ON THE ANGULAR RESOLUTION OF
SYNTHETIC APERTURE RADAR SYSTEMS

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Inglewood, California

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ABSTRACT

The influence of random phase errors on the angular resolution of a focused synthetic aperture radar system is treated. The principal measure of performance has been taken as the mean envelope power at the system output. This system output power is evaluated exactly, although not in closed form, based on the following reasonable assumptions: (1) the real beam pattern is Gaussian; (2) the random phase error is essentially a geometry independent ergodic process with a Gaussian amplitude distribution and zero mean; and (3) the random phase error has a Gaussian correlation function.

The curves presented in this report can be used to estimate expected system power response, expected system resolution, and effective aperture length beyond which, in the presence of phase error, little gain in resolution is expected.

It was found that multiple sources of error with different correlation intervals make explicit solution of the integral equation for system power response practically impossible. In this situation, a reasonable approach is to evaluate the system power response separately for each error. If one of the errors is clearly dominant, it may be regarded as bounding achievable performance.

CONTENTS

INTRODUCTION

Previous analyses regarding the effects of random phase errors on focused synthetic aperture radar systems[1,2] have been found deficient for purposes of radar resolution determination and system design.

Reference 1 presents an approximate (Monte Carlo) analysis of the expected system transfer function for various conditions of mean-square phase error and phase-error correlation length for an arbitrarily assumed exponential correlation function. Certain shortcomings exist in the analysis. First, the range of parameters is restricted to investigation of system response in the vicinity of the 3-db beamwidth. Second, many calculations are made by approximate analyses which the author finds difficult to relate to system resolution, e. g., beam canting, main lobe to sidelobe power, and 3-db beamwidth.

This author maintains that the 3-db beamwidth has questionable significance in resolution of a radar system. When expected beamwidth is used to estimate resolution,[3] then beamwidth at the limits of system dynamic range (20 to 30 db) is the significant parameter. In other words, when receiving a strong target at maximum system input, the main question is how close a target of minimum system input can be placed and its presence still be resolved. This latter definition of resolution is obviously the most stringent because, as will be shown later, the 20- to 30-db beamwidth expands very rapidly with phase errors, while the 3-db beamwidth is comparatively insensitive to these errors.

The high sensitivity of the 20- to 30-db beamwidth to phase errors allows fairly accurate estimates of system resolution from the expected power output alone. Considering the accuracy of phase-error data, use of statistical decision theory is unnecessary for all practical purposes. This is fortunate because an exact analysis of the present highly nonlinear problem using statistical decision theory would be a formidable undertaking, to say the least.

Reference 2 carries out what is essentially an exact analysis of a purported measure of beamwidth: the radius of gyration of the system output. It is clear, however, for a commonly considered system response, the $\sin^2 x / x^2$ function,[4] that no radius of gyration exists that is even remotely connected with the first lobewidth of the system output. A measure of resolution as sensitive as this to amplitude weighting of the received data is clearly dangerous to apply and cannot give a true measure of system resolution. The criterion used in Ref. 2 is consequently discarded, and its application to synthetic aperture radar resolution is considered invalid.

An omission in all previous analyses of synthetic aperture radar systems has been the determination of array length for a specified resolution in the presence of random phase errors. In the author's estimation, this quantity is the heart of system design. It is unreasonable to build a processor, storage facility, etc, compatible with a 4000-foot array when (due to irreducible random phase errors) nearly the same resultant resolution is achieved with a 2000-foot array.

Considering the foregoing, it is worthwhile to re-attack this problem from a new perspective. It is not anticipated that the approach in this report is the final answer, but the results obtained from it are considered to be of greater utility in system design.

Certain reasonable physical assumptions are made as to the character of the system in order to obtain exact mathematical solutions. The parameter calculated is the expected system average power output. Exact curves are presented to allow the system engineer to judge for himself the resultant resolution. Since the more logical 20- to 30-db resolution is highly sensitive to phase errors, the results obtained using a simple and somewhat arbitrary resolution estimate based on power response alone are not far from those that would be calculated using an exact statistical decision theory approach.

II. ANALYSIS

A vast amount of literature is available on synthetic aperture radar systems. No duplication or repetition of these works will be attempted in this research report. It is assumed that the reader is thoroughly familiar with the contents of Ref. 5 and all papers referenced therein.

This report deals only with the fully coherent focused system. A number of assumptions are made to simplify analytical study of the system.

1) Sampling rate of the radar is sufficiently high that no angular ambiguities appear in the data and also high enough that

received signal spectrum splatter due to very short term random phase fluctuations can be accommodated. This assumption allows low-pass filtering of the coherently detected received pulses to yield a CW Doppler history of a target. Continuous waveforms are much easier to treat mathematically and their use results in no loss of generality.

2) Range ambiguities for particular system geometries are suppressed by proper signal design. No further consideration of ambiguities will be treated.

3) Linearity of the radar system is assumed so that system output to any arbitrary field of targets can be obtained by voltage summation of the individual target responses.

4) A single dominant target is being viewed in each resolution cell.[6] Many targets of approximately the same return strength within one resolution cell may be approximately handled by assuming that the system output is a Rayleigh distribution with some mean power level. A small number of targets of approximately equal strength in one resolution cell can cause "break up" of the return due to cancellation and reinforcement. This is one of the reasons for the different appearance of optical and radar maps.

Consider now Fig. 1, which depicts the system geometry while responding to a single point target. It is assumed that an observer at 0 fixed relative to the real antenna watches the target, T, go by with velocity, v. Time is

referenced to zero as indicated in the figure. The observer and his data

processor are "matched" to the geometry of Fig. 1. Any change in the true

situation from that of Fig. 1 due to improper target (vehicle) motion, clock

(oscillator) instabilities, or propagation-medium scintillation must

be compensated for by either a change in data processing or by accepting the

change as random error, with the resulting system degradation. This report

is concerned only with uncompensated phase errors and their effects on

angular resolution.

Let the radar at time, t, transmit the signal

$$V_t = E_t \cos \omega t \qquad (1)$$

where E_t = transmitted signal amplitude, volts

ω = carrier radian frequency, rad/sec

Assuming Gaussian real antenna beam patterns, it is a simple matter

to show that the return signal from a point target is given to first order by

$$V_r = E_r \exp(-2kt^2) \cos[\omega t + \alpha + \beta t^2 + \phi_i(t)] \qquad (2)$$

where E_r = received signal amplitude, volts

k = beam factor, $\sec^{-2}$

ω = carrier radian frequency, rad/sec

α = fixed phase shifts which may exist to and from target, e.g.,

$2R/c$ plus propagation and equipment biases.

and β = coefficient for change in phase versus time due to quadratic

range change (Fig. 1), $\sec^{-2}$

$\phi_i(t)$ = i^{th} ensemble member of an ergodic random Gaussian phase

process of zero mean and Gaussian correlation function, rad

It is not necessary to know the value of a as it will cancel in later calcula-
tions.   The terms β and k may be computed from very simple geometric
considerations.   The results are

$$\beta = -\frac{2\pi v^2}{\lambda R} \;,\; \sec^{-2} \tag{3}$$

$$k = 2\left(\frac{v}{L}\right)^2 \;,\; \sec^{-2} \tag{4}$$

where   L = length at the target to the 1/e (-4.34 db) points of the real

antenna one-way power pattern, ft

v = vehicle velocity, ft/sec

λ = carrier wavelength, ft

R = range to target at t = 0 (See Fig. 1), ft

For convenience, β and k are used in the following analysis to minimize
symbology.

The synthetic aperture radar system correlates the received signal,
given by Eq. (2), with a delayed in-phase and quadrature replica and pre-
sents an average power output equal to the sum of the squares of the in-phase
and quadrature correlator responses.   Figure 2 depicts this process.   The

limits of integration are shown to be infinite in Fig. 2. In a real system, a

finite integration from $\approx$ -L/v to L/v would be accomplished. Since mathe-

matical tractability is sought and, further, since, with the assumed Gaussian

weighting, there is little difference in the correlator output if L/v or the

infinite limits are used, the analysis to follow uses infinite limits.

As depicted, the correlators are "matched" filters for the signal of

Eq. (2) and, as such, maximize the signal-to-noise ratio in the output for a

time shift $\Delta = 0$. However, there are occasions when all the data received

from a target on its pass through the real beam cannot and should not be

processed. These situations occur when (1) processing time is precious and

resolutions corresponding to real beam length, L, are not needed, (2) lack

of phase coherence across length, L, yields a point of diminishing returns

and processing more data gives little increase in system resolution, and

(3) vehicle constraints have forced the use of a smaller real antenna than

desirable from a resolution point of view.

As a consequence of the above, the reference functions chosen for

analysis are generalized to the following

$$\exp\left[-2k\left(\frac{t-\Delta}{\eta}\right)^2\right] \cos\left[\omega t + \beta(t-\Delta)^2 + \gamma\right] \tag{5}$$

$$\exp\left[-2k\left(\frac{t-\Delta}{\eta}\right)^2\right] \sin\left[\omega t + \beta(t-\Delta)^2 + \gamma\right] \tag{6}$$

where η = (by definition) the fraction of the data utilized in processing,
$0 \le \eta \le 1$.

η, as depicted in Eq. (5) and Eq. (6) may be greater than 1 and not necessarily restricted to the range $0 \leq \eta \leq 1$. However, in the presence of noise, the signal-to-noise ratio at the correlator outputs will drop off for $\eta > 1$.[7] Since for every $\eta > 1$ there is a $\eta < 1$ with the same signal-to-noise ratio, it is obvious from an engineering point of view that the range of η resulting in the least data processing will always be used; ergo, $0 \leq \eta \leq 1$.

Having dispensed with all the assumptions and system descriptions, injection of the voltage given by Eq. (2) into the system of Fig. 2 with the reference functions of Eq. (5) and (6) gives the following average power output

$$P_o = \overline{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[\exp\left\{-2k\left[x^2 + y^2 + \left(\frac{x - \Delta}{\eta}\right)^2 + \left(\frac{y - \Delta}{\eta}\right)^2\right]\right\}\right]} \\ \times \overline{\cos\left[2\Delta\beta(x - y) + \phi_i(x) - \phi_i(y)\right]} \, dxdy \tag{7}$$

The bar over Eq. (7) is the ensemble average over ϕ_i. Since the average of a sum is the sum of the averages and, further, since $\overline{\sin\left[\phi_i(x) - \phi_i(y)\right]} \equiv 0$ and $\overline{\phi_i(x) - \phi_i(y)} \equiv 0$, expansion of the cosine function in Eq. (7) yields

$$P_o = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[\exp\left\{-2k\left[x^2 + y^2 + \left(\frac{x - \Delta}{\eta}\right)^2 + \left(\frac{y - \Delta}{\eta}\right)^2\right]\right\}\right] \\ \times \cos\left[2\Delta\beta(x - y)\right] \overline{\cos\left[\phi_i(x) - \phi_i(y)\right]} \, dxdy \tag{8}$$

Since $\phi_i(t)$ has been assumed to be Gaussian, it is recognized immediately[8]
that

$$\overline{\cos\left[\phi_i(x) - \phi_i(y)\right]} \cong \exp\left\{\frac{-\overline{\left[\phi_i(x) - \phi_i(y)\right]^2}}{2}\right\} \qquad (9)$$

Simplifying Eq. (9) we get

$$\overline{\cos\left[\phi_i(x) - \phi_i(y)\right]} = \exp\left\{-[R(0) - R(x - y)]\right\} \qquad (10)$$

where $R(x - y)$ = correlation function of the random phase
perturbations, rad^2

Let us assume now $R(x - y)$ is Gaussian in nature, thus

$$R(x - y) = \sigma^2 \exp\left[-\frac{(x - y)^2}{\tau_c^2}\right] , \ rad^2 \qquad (11)$$

where σ^2 = mean square value of phase error, rad^2

τ_c = time it takes the correlation function of phase to drop to
1/e of its zero argument value, sec

Substituting Eq. (11) into Eq. (10) and Eq. (10) into Eq. (8), we finally
obtain

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$$P_0 = \exp(-\sigma^2)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[\exp\left\{ -2k\left[x^2 + y^2 + \left(\frac{x-\Delta}{\eta}\right)^2 + \left(\frac{y-\Delta}{\eta}\right)^2 \right] \right. \right.$$

$$\left. \left. + \sigma^2 \exp\left[-\frac{(x-y)^2}{\tau_c^2} \right] \right\} \right] \times \cos\left[2\Delta\beta(x-y) \right] dx\,dy \quad (12)$$

Our task now is to evaluate the interesting integral of Eq. (12). Expanding the $\exp\left\{ \sigma^2 \exp\left[-(x-y)^2/\tau_c^2 \right] \right\}$ term in an infinite series, which is valid for all values of the exponent, Eq. (12) becomes

$$P_0 = \exp(-\sigma^2)\sum_{j=0}^{\infty}\frac{\sigma^{2j}}{j!}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[\exp\left\{ -2k\left[x^2 + y^2 + \left(\frac{x-\Delta}{\eta}\right)^2 + \left(\frac{y-\Delta}{\eta}\right)^2 \right] \right. \right.$$

$$\left. \left. -\frac{j(x-y)^2}{\tau_c^2} \right\} \right] \cos\left[2\Delta\beta(x-y) \right] dx\,dy \quad (13)$$

Expanding the exponent in Eq. (13) and performing the excruciating double integration with the help of integral tables[9] give the following <u>exact</u> result for P_0

$$P_0' = \exp(-\sigma^2)\sum_{j=0}^{\infty} \frac{\sigma^{2j}\exp\left[-4\left(\frac{x}{x_0}\right)^2 \left\{ \left[1 + \dfrac{j}{4\left(\frac{x_c}{L_{eff}}\right)^2} \right]^{-1} + 4\left(\frac{x_0}{L_{eff}}\right)^2 \dfrac{\eta^2}{(\eta^2+1)^2} \right\} \right]}{j!\left[1 + \dfrac{j}{4\left(\frac{x_c}{L_{eff}}\right)^2} \right]^{1/2}} \quad (14)$$

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where P_0' = normalized system power response

 x = distance along vehicle track, ft

 x_0 = $(2/\pi)(\lambda R)/L_{eff}$, - 4.34 db (1/e) synthetic aperture

response width, under no phase-error conditions, ft

 $L_{eff} = L\left(\dfrac{2}{1 + 1/\eta^2}\right)^{1/2}$, by definition, the effective synthetic

aperture length, ft

 $x_c = v\tau_c$, the distance to 1/e point of the phase error

correlation function, ft

 η = (by definition) the fraction of available data processed.

See Eq. (5) and Eq. (6).

Two key quantities defined above bear repeating as Eq. (15) and
Eq. (16).[10]

$$x_0 = \frac{2}{\pi}\left(\frac{\lambda R}{L_{eff}}\right) \quad , \ ft \quad (15)$$

$$L_{eff} = L\left(\frac{2}{1 + 1/\eta^2}\right)^{1/2} \quad , \ ft \quad (16)$$

Note that, for small η, Eq. (14) reduces identically to the two-way power
pattern of the real beam. This is expected since, for small η, little proc-
essing of the received data is performed.

For the case of considerable beam sharpening, $x_0/L_{eff} \to 0$, the exact equation, Eq. (14), reduces to

$$P_0' = \exp(-\sigma^2) \sum_{j=0}^{\infty} \frac{\sigma^{2j} \exp\left\{-4\left(\dfrac{x}{x_0}\right)^2 \left[1 + \dfrac{j}{4\left(\dfrac{x_c}{L_{eff}}\right)^2}\right]^{-1}\right\}}{j! \left[1 + \dfrac{j}{4\left(\dfrac{x_c}{L_{eff}}\right)^2}\right]^{1/2}} \qquad (17)$$

Note that in Eq. (17) the power response of the system depends only on (1) σ, the rms magnitude of the random phase error, (2) x_c/L_{eff}, the ratio of the correlation distance of phase errors to effective array length, and (3) x/x_0, the normalized distance from maximum output. Note also, as $x_c/L_{eff} \to \infty$ (complete correlation across the array), Eq. (17) becomes

$$P_0' = \exp\left[-4\left(\frac{x}{x_0}\right)^2\right] \qquad (18)$$

and is completely uninfluenced by the rms magnitude of the phase error. This is expected. Equation (18) demonstrates the significance of x_0 as the width to the $1/e$ (-4.34 db) power response with no phase error.

Figures 3(a) - (n) are plots of Eq. (17) for various values of x_c/L_{eff} and σ. Neglect of the last term in Eq. (14) for the parameters chosen yields

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an overestimate of "sidelobe" power[11] of ≈ 0.1 db at $x/x_0 = 5$ and exponentially less error for smaller x/x_0.

Figures 4(a) - (e) are interesting and can assist in assessing the effective array length one should use in the presence of phase errors to achieve a specified 20 db synthetic beamwidth. These curves are normalized and can be used for any range R or system wavelength λ. For the special case of $R = R_0 = 1.8 \times 10^6$, ft, and $\lambda = \lambda_0 = 0.1$, ft, the 20 db half-beamwidth appears directly as the ordinate.

Note the rapid expansion with phase error of the 20-db beamwidth as contrasted to the 3-db beamwidth. A reasonable definition of resolution is the half-beamwidth to the point equal in decibels to the dynamic range of the system. For N strong targets in the vicinity and for short phase-error correlation distances, it is suggested that a safety factor of $10 \log_{10}(N)$ be added to the dynamic range [see Fig. 3(a)]. It is felt that this is a good engineering criterion considering (1) the sensitivity of the dynamic-range beamwidth to σ and x_c, and, (2) the lack of precise knowledge of σ and x_c.

III. CONCLUSIONS

For the first time, reasonable assumptions regarding the physical system and random phase errors have yielded an exact result that predicts the expected power response for a synthetic aperture radar system.

Complete freedom to choose the fraction of the received data to be processed has been left to the system designer by his choice of η.

The assumed Gaussian weighting not only yielded a mathematically trac-
table system but, in addition, gave an expected response with no sidelobes.

The definition of resolution suggested by the author is the half-beamwidth
to the dynamic range point on the expected power response curve. A figure
of 20 db was chosen for illustration in Fig. 4(a) - (e). The definition of reso-
lution as the dynamic-range half-beamwidth is consistent with (1) the high
sensitivity of resolution to σ and x_c and (2) the inaccurate knowledge of
these parameters.

It is felt that an exact decision theory attack on this problem would not
be fruitful because of its mathematical formidability and the inaccurate
available data on σ and x_c.

Equations (11) and (12) indicate that multiple sources of error with
different correlation intervals make explicit solution of the integral equation
for system power response a formidable undertaking. In this situation, a
reasonable approach is to evaluate the system power response separately
for each error. If one of the errors is clearly dominant, it may be regarded
as bounding achievable performance.

REFERENCES

[1]C. A. Greene and R. T. Moller, "The Effect of Normally Distributed Random Phase Errors on Synthetic Array Gain Patterns," IRE Trans. on Military Electronics vol. MIL-6, no. 2, 130-139; April 1962.

[2]W. M. Brown and C. J. Palermo, "Effects of Phase Errors on the Ambiguity Function," IEEE International Convention Record; March 1963.

[3]A reasonable measure due to its analytical tractability.

[4]Constant amplitude weighting over the effective array length.

[5]IRE Trans. On Military Electronics vol. MIL-6, no. 2; April 1962.

[6]If a dominant target exists and is surrounded by much smaller targets all within one resolution cell, the effect of these smaller targets may be approximately characterized as random phase variations on the return of the dominant target.

[7]Since only a matched filter, $\eta = 1$, can yield the largest signal-to-noise ratio for $\Delta = 0$.

[8]J. A. Develet, Jr., "A Threshold Criterion for Phase-Lock Demodulation," Proc. IEEE, vol. 51, 350; February 1963.

[9]W. Grobner and N. Hofreiter, "Integraltafel, Zweiter Teil, Bestimmte Integral," Wien und Innsbruck, Springer-Verlag, 1958; p. 143, Equations 1a,b.

[10]Note, in Eq. 14-16, the time variable along the track has been replaced by the more conventional distance variable.

REFERENCES (Continued)

[11]Note with Gaussian weighting no sidelobes exist at all! Therefore, if
sidelobe suppression is important, Gaussian amplitude weighting is indicated.

FIGURES

Fig. 1.   System Geometry

$$\frac{2}{E_r}e^{-2k(t-\Delta)^2}\cos\left[\omega t + \beta(t-\Delta)^2 + \gamma\right]$$

$$\frac{2}{E_r}e^{-2k(t-\Delta)^2}\sin\left[\omega t + \beta(t-\Delta)^2 + \gamma\right]$$
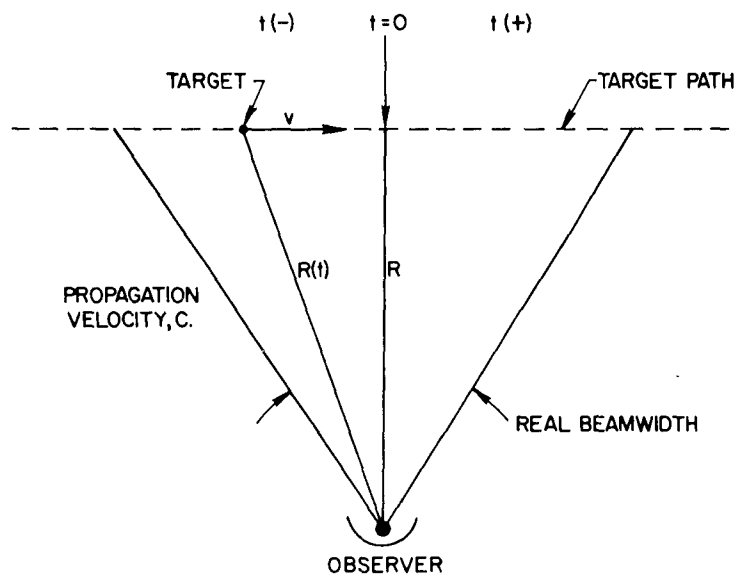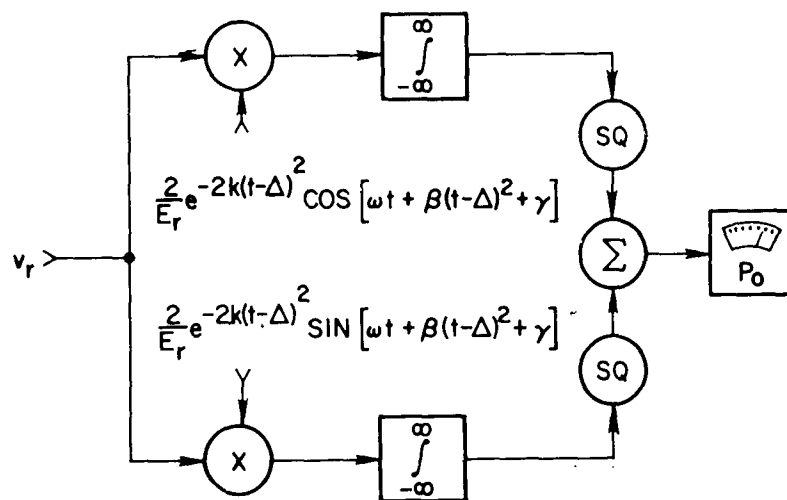
Fig. 2.   System Transfer Function

(a)

Fig. 3.   System Power Response Versus
Distance Along Vehicle Track

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(b)

Fig. 3.   System Power Response Versus
Distance Along Vehicle Track

NORMALIZED SYSTEM RESPONSE, db

0
−10
−20
−30
−40
−50
−60

0   1.0   2.0   3.0   4.0   5.0

DISTANCE ALONG VEHICLE TRACK, X/X₀

100 DEG
120 DEG
80 DEG
60 DEG
180 DEG
40 DEG
20 DEG
10 DEG
σ = 5 DEG

$$\frac{X_c}{L_{eff}} = 0.04$$
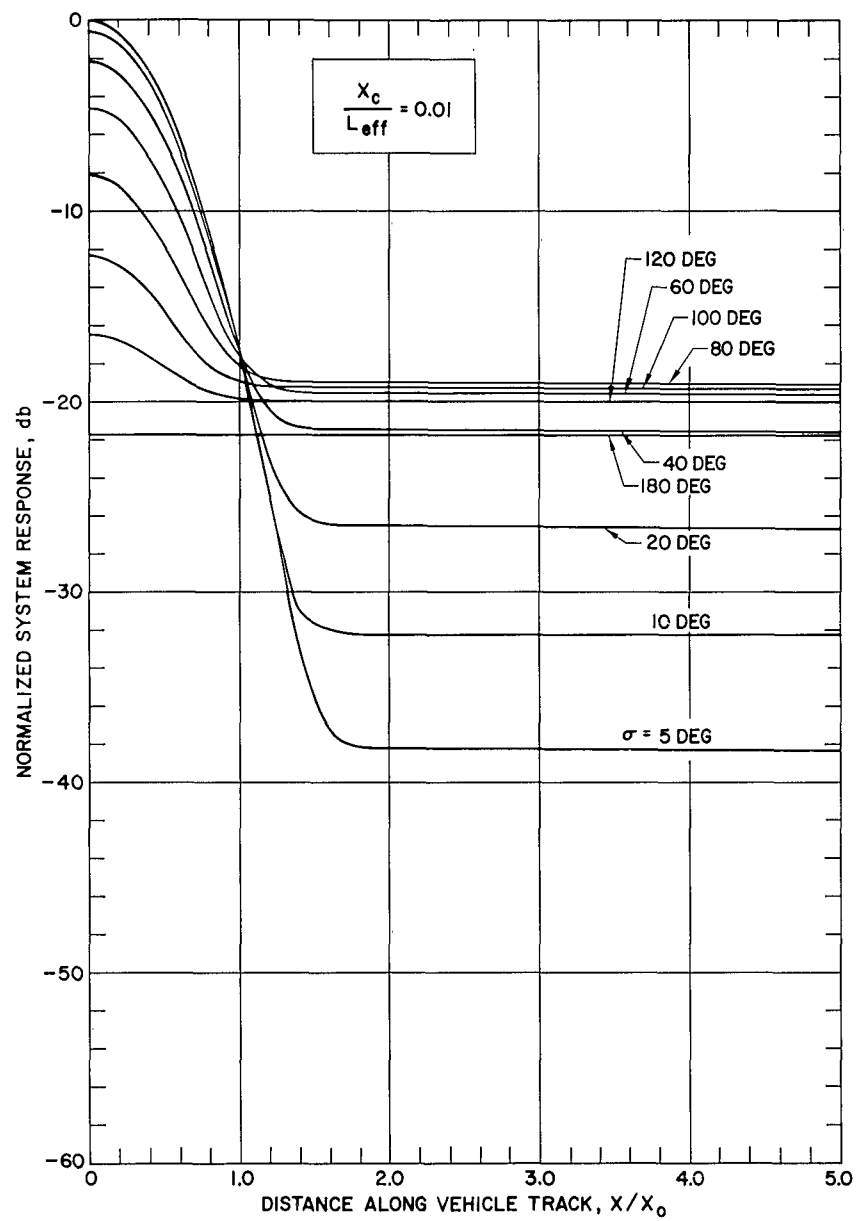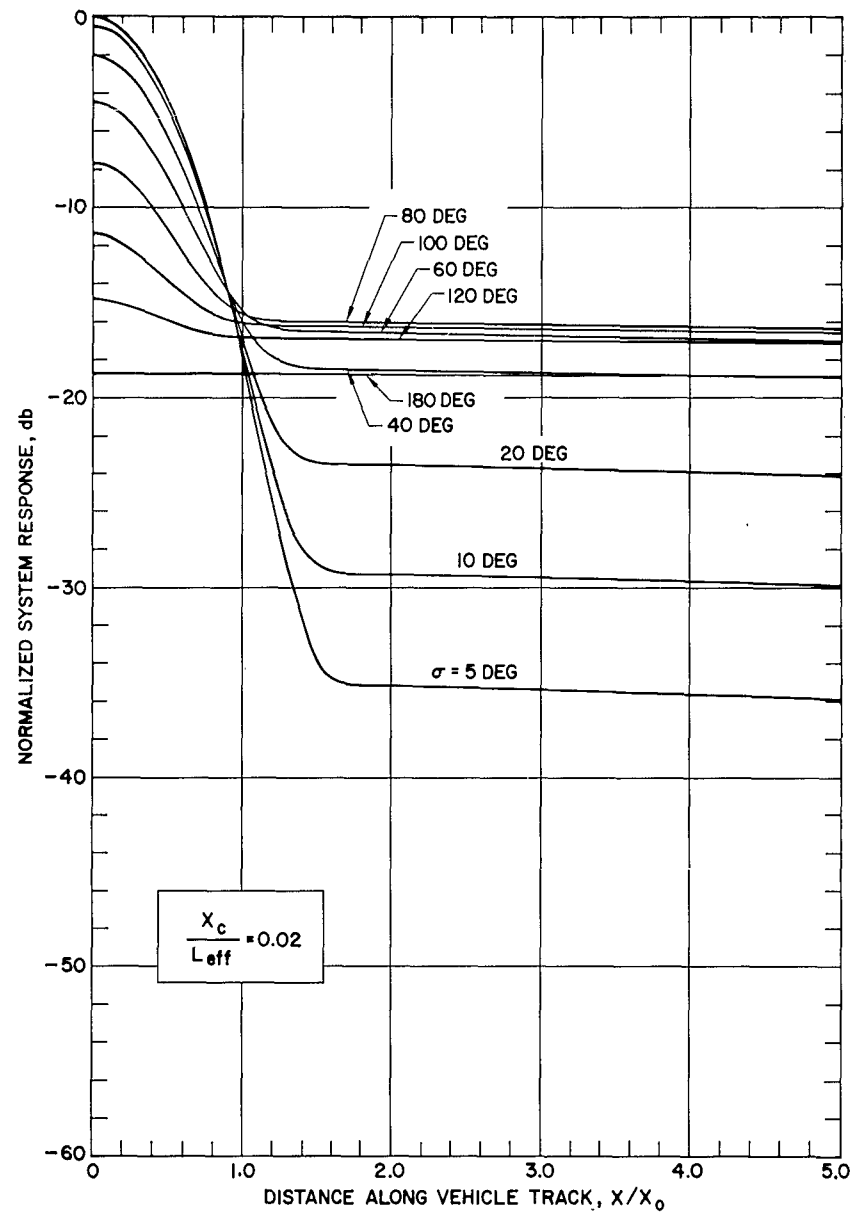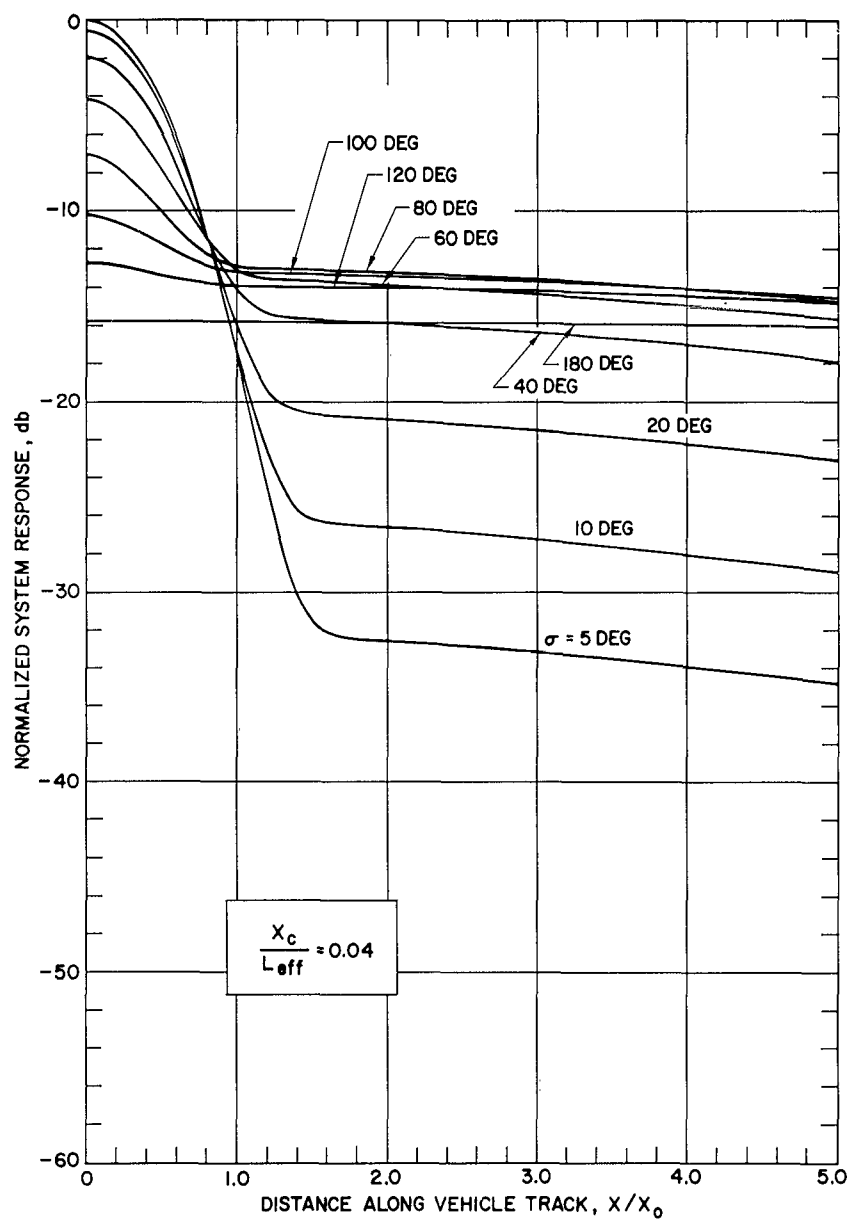
(c)

Fig. 3. System Power Response Versus
Distance Along Vehicle Track

-24-
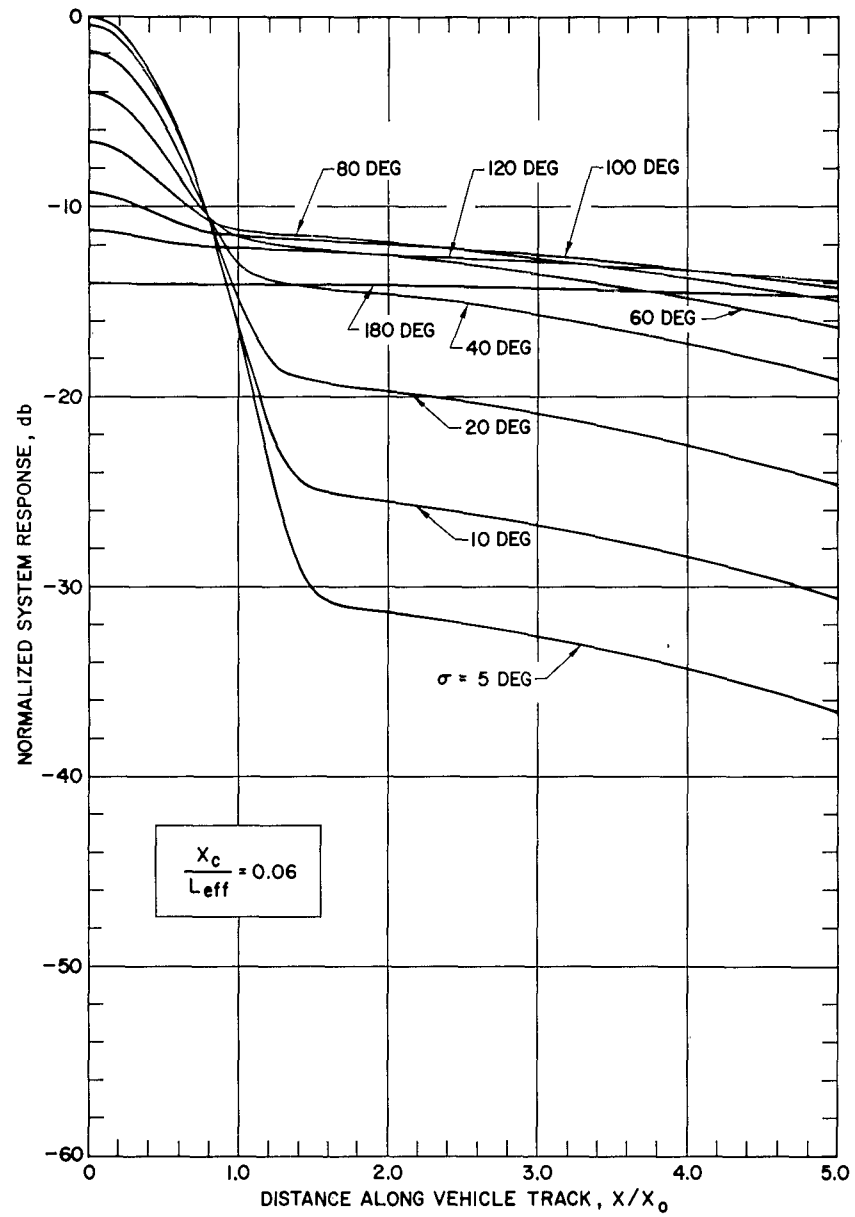
(d)

Fig. 3. System Power Response Versus
Distance Along Vehicle Track
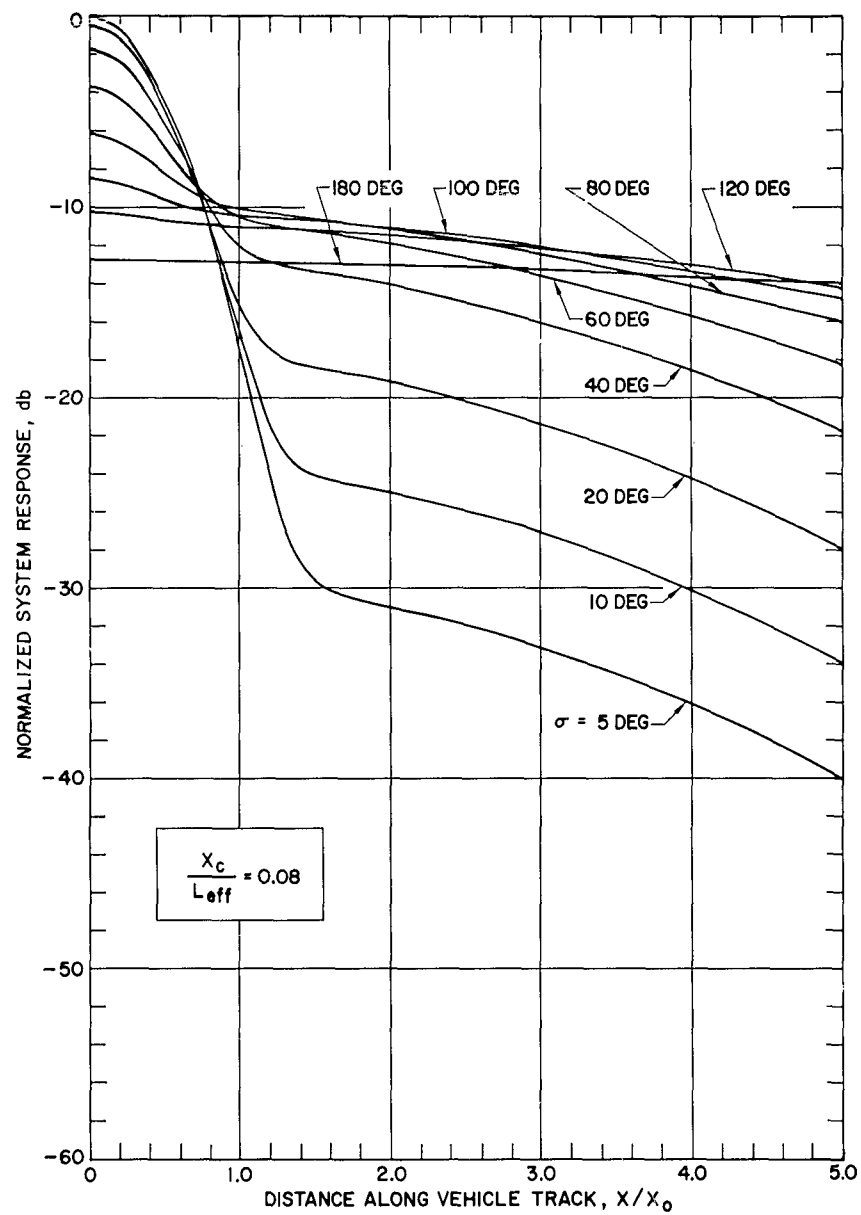
(e)

Fig. 3.   System Power Response Versus
Distance Along Vehicle Track
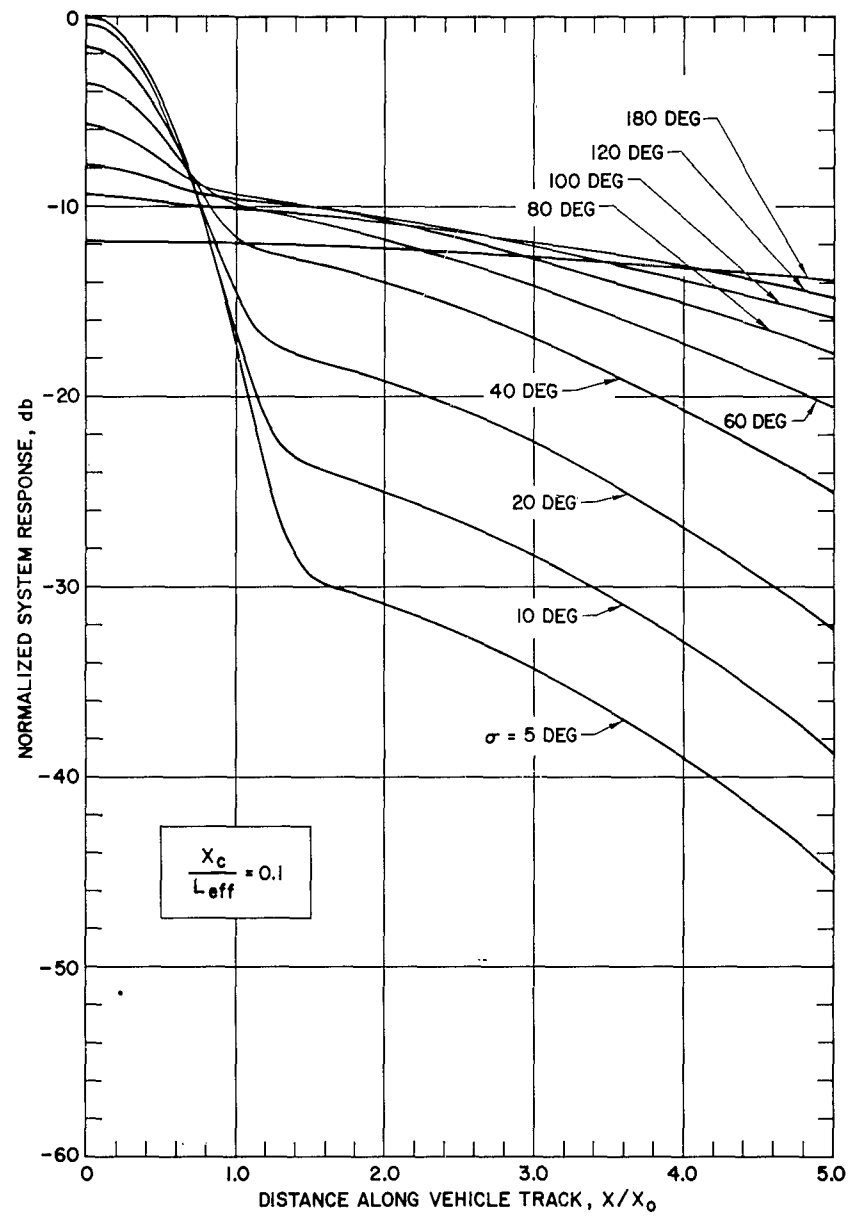
(f)

Fig. 3.   System Power Response Versus
Distance Along Vehicle Track
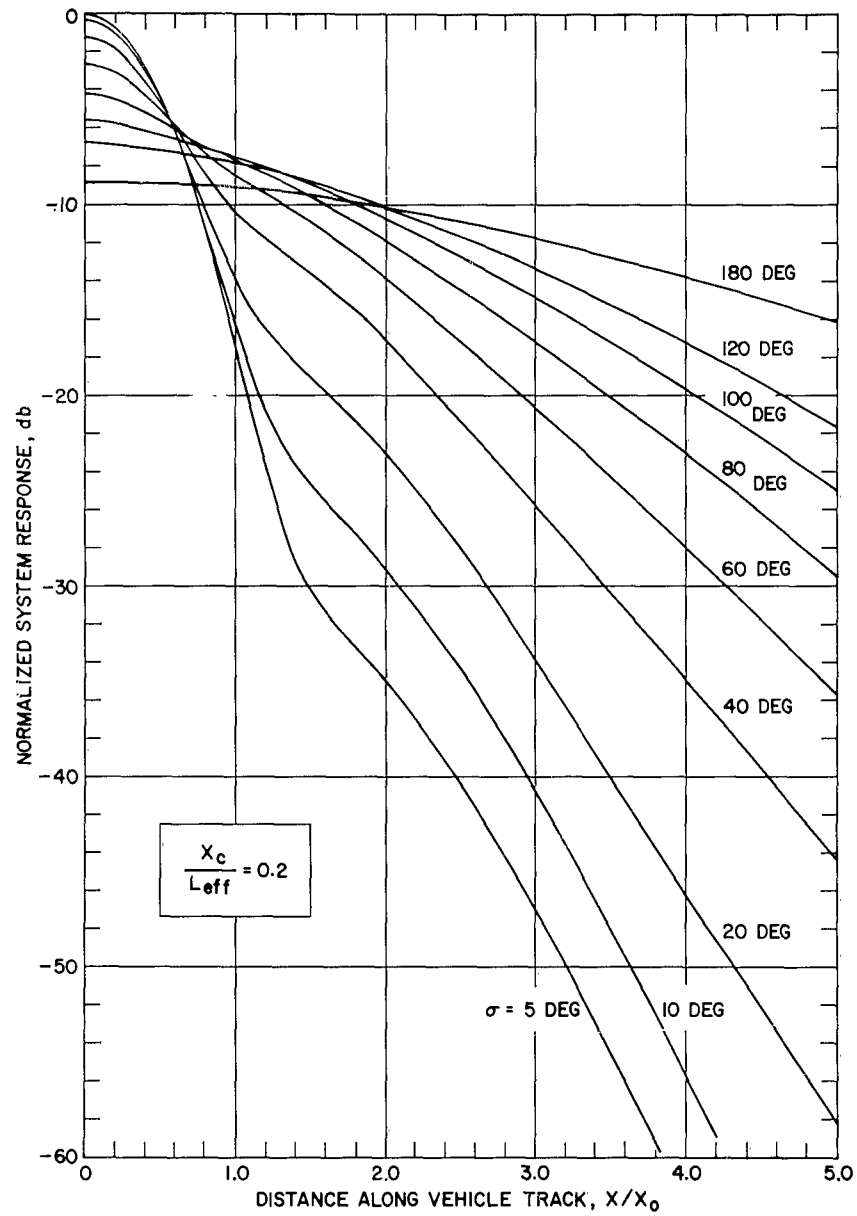
(g)

Fig. 3. System Power Response Versus
Distance Along Vehicle Track
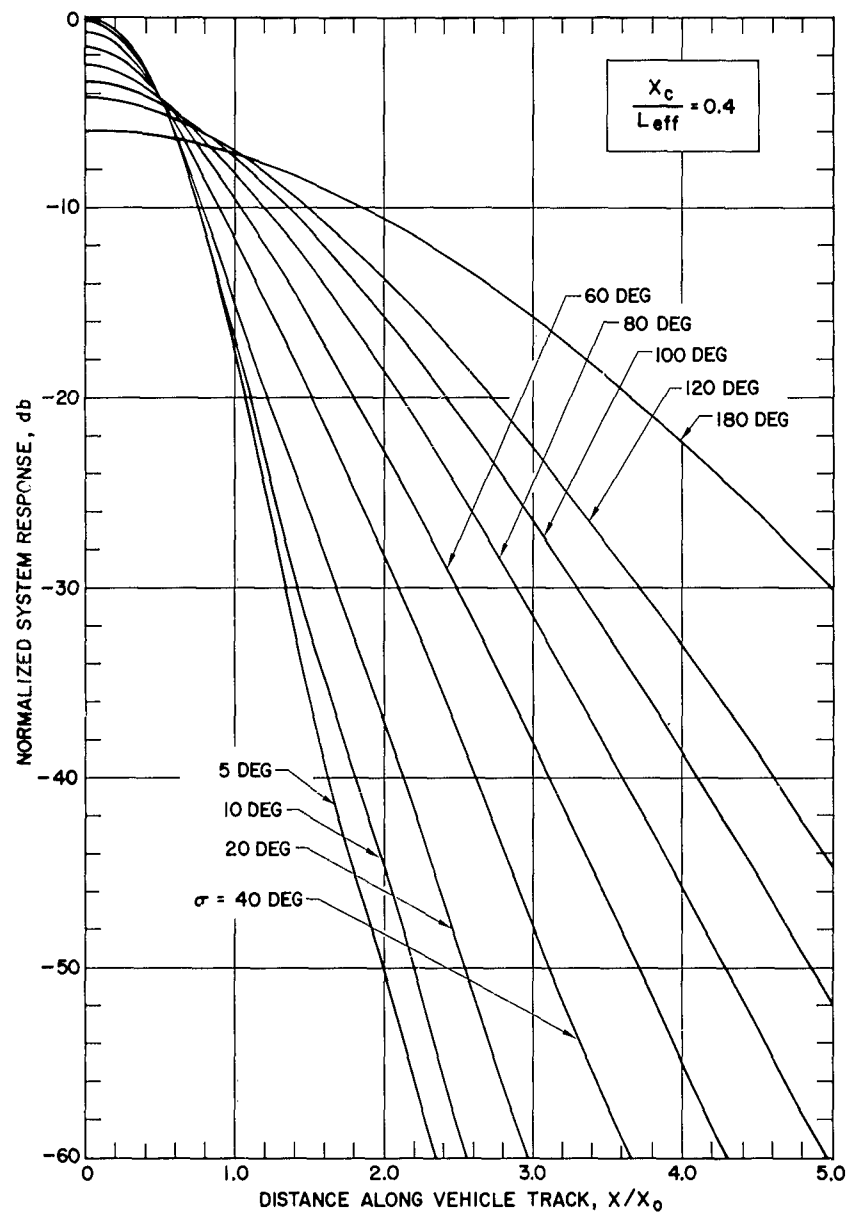
-28-

Fig. 3.   System Power Response Versus
Distance Along Vehicle Track
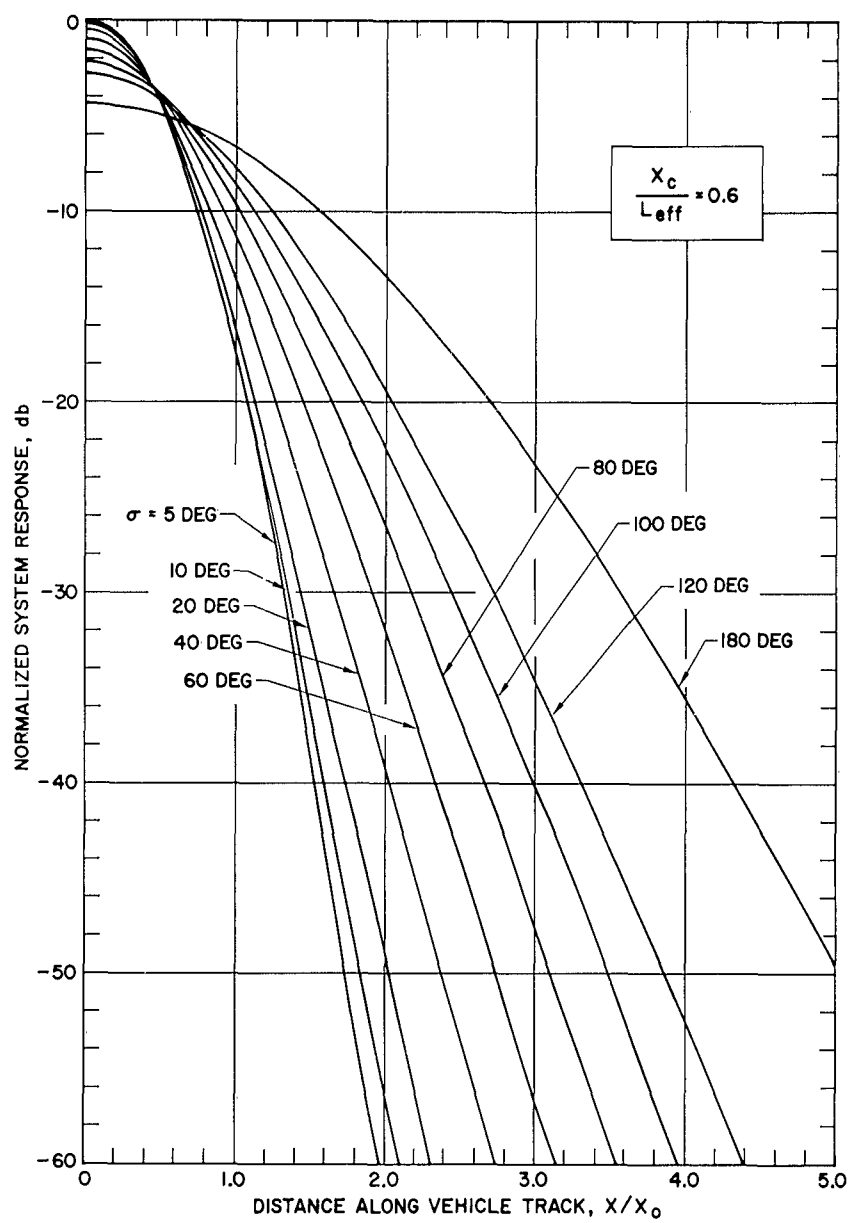
Fig. 3. System Power Response Versus
Distance Along Vehicle Track
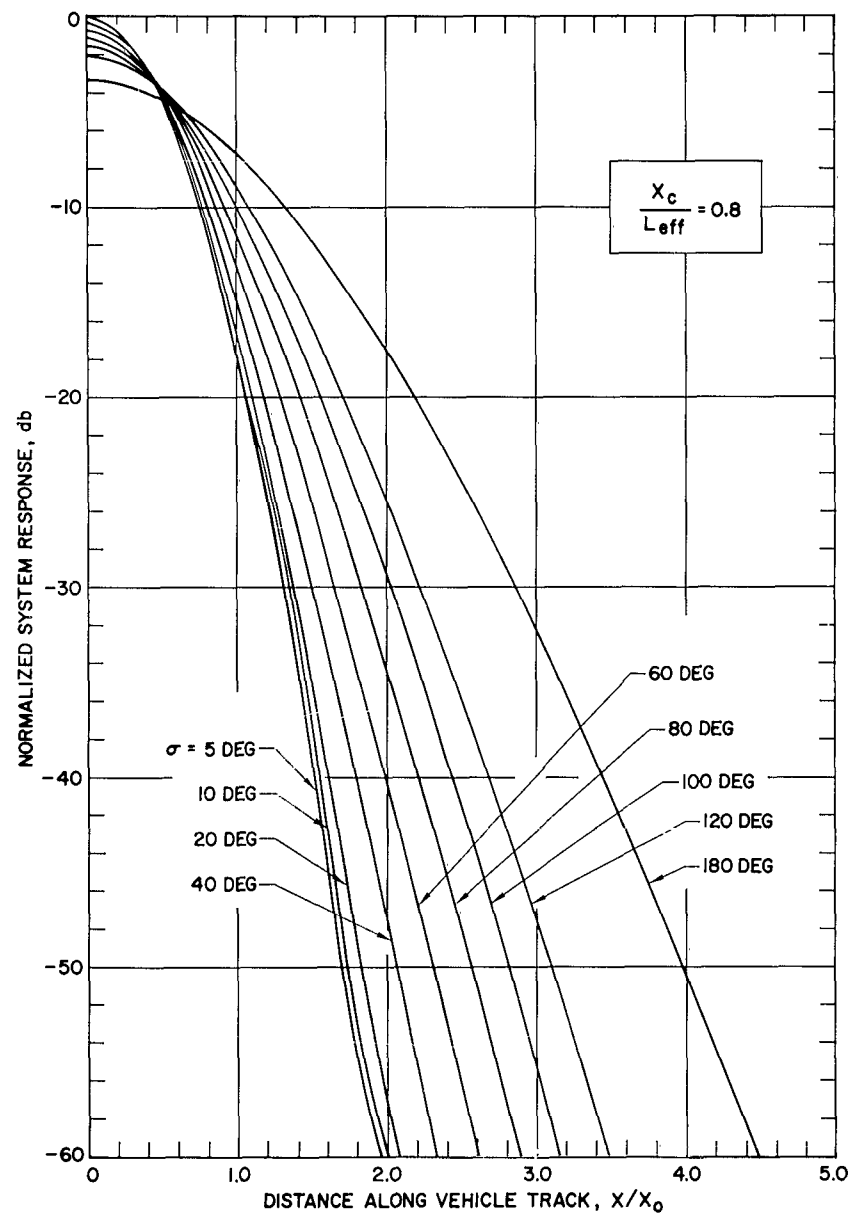
Fig. 3.   System Power Response Versus
Distance Along Vehicle Track

(k)
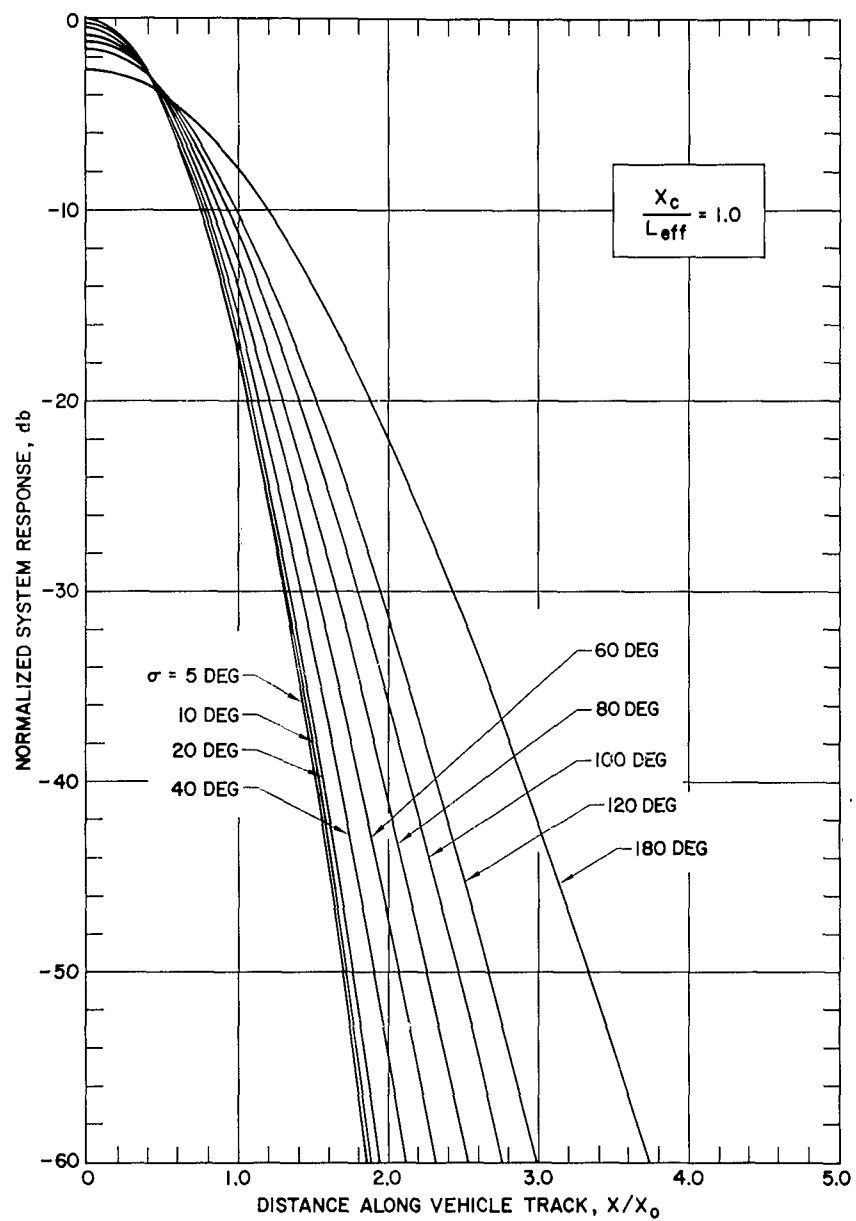
Fig. 3.   System Power Response Versus
Distance Along Vehicle Track
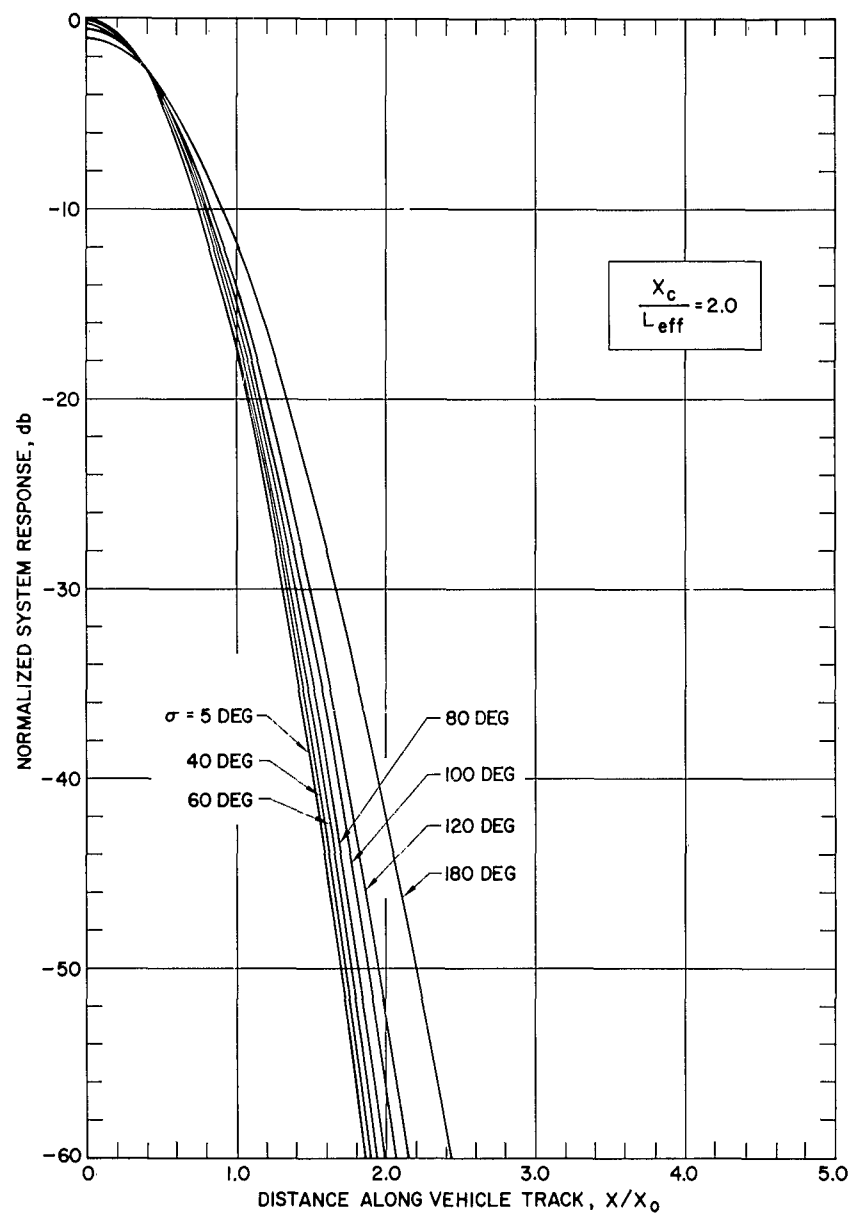
(1)

Fig. 3. System Power Response Versus
Distance Along Vehicle Track
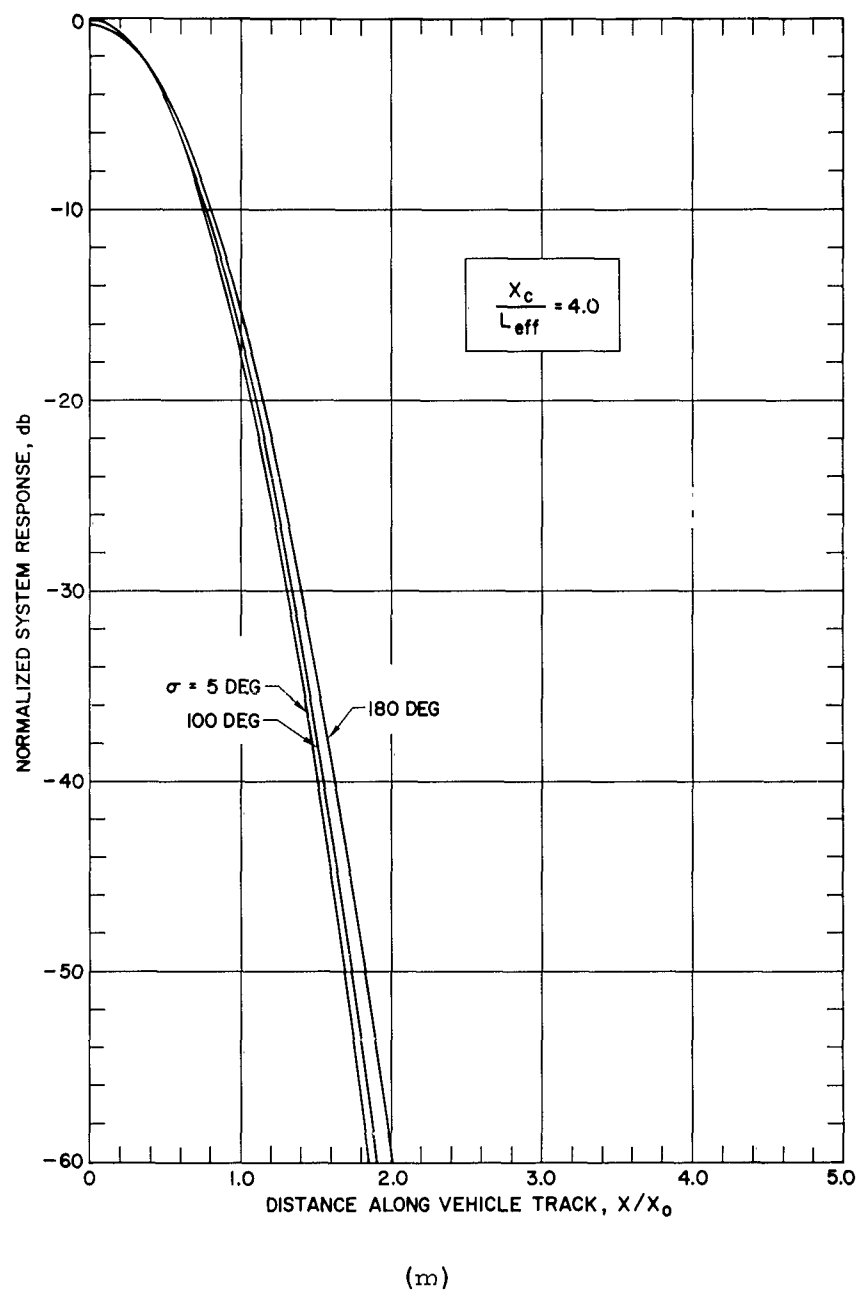
Fig. 3. System Power Response Versus
Distance Along Vehicle Track

(n)

Fig. 3. System Power Response Versus
Distance Along Vehicle Track

Fig. 4. Twenty-Decibel Half-Beamwidth Versus Array Length

(a)

Fig. 4. Twenty-Decibel Half-Beamwidth Versus Array Length

EFFECTIVE ARRAY LENGTH, L_{eff}, ft

(c)

NORMALIZED 20 db HALF BEAMWIDTH, $X_{20}\left(\frac{\lambda_0}{\lambda}\right)\left(\frac{R}{R_0}\right)$, ft

$\sigma = 180$ DEG

120 DEG
100 DEG
80 DEG

60 DEG
40 DEG
20 DEG
10 DEG
5 DEG

$X_c = 1000$ ft
$R_o = 1.8 \times 10^6$ ft
$\lambda_o = 0.1$ ft

Fig. 4.   Twenty-Decibel Half-Beamwidth Versus Array Length

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Fig. 4.   Twenty-Decibel Half-Beamwidth Versus Array Length

Fig. 4. Twenty-Decibel Half-Beamwidth Versus Array Length

(e)

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Aerospace Corporation, El Segundo, California.
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ON THE ANGULAR RESOLUTION OF SYNTHETIC
APERTURE RADAR SYSTEMS. Prepared by Jean
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20 June 1963. [52] p. incl. illus.
(Report TDR-169(3250-43)TN-4; SSD-TDR-63-130)
(Contract AF 04(695)-169)   Unclassified Report

The influence of random phase error on the angular
resolution of a focused synthetic aperture radar
system is treated. The principal measure of per-
formance has been taken as the mean envelope
power at the system output. This system output
power is evaluated exactly, although not in closed
form, based on the following reasonable assump-
tions: (1) the real beam pattern is Gaussian;
(2) the random phase error is essentially a geom-
etry independent ergodic process with a Gaussian
amplitude distribution and zero mean; and (3) the
(over)

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random phase error has a Gaussian correlation function. The curves presented in this report can be used to estimate expected system power response, expected system resolution, and effective aperture length beyond which, in the presence of phase error, little gain in resolution is expected. It was found that multiple sources of error with different correlation intervals make explicit solution of the integral equation for system power response practically impossible. In this situation, a reasonable approach is to evaluate the system power response separately for each error. If one of the errors is clearly dominant, it may be regarded as bounding achievable performance.

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